Classical hydrodynamics, however, does not afford a satisfactory means of dealing with the general equations of motion derived above, for they involve the density, and wherever, as a consequence, it is necessary to take into account the physical properties of the fluid, the classical theory practically always assumes an equation of state of the form $f(p, \rho) = 0$, the density being a function of the pressure only.[6] In the actual cases of nature, particularly in meteorology and hydrography, many other independent variables enter, such as temperature, humidity, salinity, etc. The hydrodynamical theory of *baroclinic* fluids—i. e., fluids in which other independent variables than the pressure also affect the density—has been worked out by Bjerknes and has recently been made easily accessible in elementary form by Appell.[7] In such fluids surfaces of equal density are not always surfaces of equal pressure, and the formation and annihilation of vortices are possible.

The application of this theory to the dynamics of the earth's atmosphere has also been largely the work of Bjerknes, having been worked out in parallel with the well-known empirical investigations of the Bergen meteorologists. There has recently appeared a comprehensive and up-to-date summary of the whole subject,[8] which constitutes a most valuable memoir on theoretical meteorology.

[6] See Lamb, *op. cit.*, art. 8; *cf.* Appell, *op. cit.*, art. 627.

[7] Appell, *op. cit.*, chap. xxxii, pp. 562–605.

[8] V. Bjerknes, On the Dynamics of the Circular Vortex, with applications to the Atmosphere and Atmospheric Vortex and Wave Motions. *Geofysiske Publikationer*, Vol. II, No. 4, Kristiania, 1921. 4to, 88 pp.

SHORT METHOD OF OBTAINING A PEARSON COEFFICIENT OF CORRELATION, AND OTHER SHORT STATISTICAL PROCESSES.

By FRANK M. PHILLIPS, Ph. D.

[U. S. Public Health Service, Washington, D. C., March 1, 1922.]

The usual method of obtaining a Pearson coefficient of correlation is somewhat long and tedious, especially if there be a large number of paired measures and if the measures or the averages of these happen to be such as to involve either large numbers or numbers running out to two or three decimal places. It is the purpose of this paper to derive and illustrate a shorter method, which at the same time will tend to eliminate errors likely to creep into a solution by the ordinary method. The formulas given in this article have all been derived by purely mathematical processes and do not involve any approximations; neither the average nor the deviations are used in computations by them; they shorten the work materially when solving for average deviation, standard deviation, coefficient of variability, and coefficient of correlation.

Let—

n = number of independent, or of paired, measures.
n_- = number of measures below the average.
n_+ = number of measures above the average.
Σm = sum of independent measures.
Σm_- = sum of measures below the average.
Σm_+ = sum of measures above the average.
S = measures of "subject."
R = measures of "relative."
a = average of the "subject."
c = average of the "relative."

Then the usual process of getting the coefficient of correlation may be represented as follows:

S	R	x	y	x^2	y^2	xy.
S_1...	R_1..	S_1-a.	R_1-c..	$S_1^2-2S_1a+a^2$....	$R_1^2-2R_1c+c^2$....	$S_1R_1-S_1c-R_1a+ac$.
S_2...	R_2..	S_2-a	R_2-c..	$S_2^2-2S_2a+a^2$....	$R_2^2-2R_2c+c_2$....	$S_2R_2-S_2c-R_2a+ac$.
S_3...	R_3..	S_3-a	R_3-c..	$S_3^2-2S_3a+a^2$....	$R_3^2-2R_3c+c^2$....	$S_3R_3-S_3c-R_3a+ac$.
.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....
S_n..	R_n.	S_n-a	R_n-c..	$S_n^2-2S_na+a^2$...	$R_n^2-2R_nc+c^2$...	$S_nR_n-S_nc-R_na+ac$.
ΣS..	ΣR	....	....	$\Sigma S^2-2\Sigma Sa+na^2$..	$\Sigma R^2-2\Sigma Rc+nc^2$.	$\Sigma SR-\Sigma Sc-\Sigma Ra+nac$.

$$r = \frac{\Sigma xy}{n\sigma_S\sigma_R} = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}}$$

Now, since $\Sigma S = na$, and $\Sigma R = nc$, then $\Sigma Sc = nac$, $\Sigma Ra = nac$, and $\Sigma SR - \Sigma Sc - \Sigma Ra + nac = \Sigma SR - nac$; furthermore, since $\Sigma Sa = na^2$, and $\Sigma Rc = nc^2$, then $\Sigma S^2 - 2\Sigma Sa + na^2 = \Sigma S^2 - na^2$, and $\Sigma R^2 - 2\Sigma Rc + nc^2 = \Sigma R^2 - nc^2$.

Therefore, we have—

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}} = \frac{\Sigma SR - \Sigma Sc - \Sigma Ra + nac}{\sqrt{(\Sigma S^2 - 2\Sigma Sa + na^2)\ (\Sigma R^2 - 2\Sigma Rc + nc^2)}}$$
$$= \frac{\Sigma SR - nac}{\sqrt{(\Sigma S^2 - na^2)\ (\Sigma R^2 - nc^2)}},$$

a formula much better adapted to numerical computation.

To illustrate the advantage of this method, let us take the following seven pairs of related measures and solve for a coefficient of correlation by the usual method:

S	R	x	x^2	y	y^2	xy
4	5	−3	9	−7	49	21
5	8	−2	4	−4	16	8
6	11	−1	1	−1	1	1
7	11	0	0	−1	1	0
8	13	1	1	1	1	1
9	18	2	4	6	36	12
10	18	3	9	6	36	18
49 $a=7$	84 $c=12$	....	28	....	140	61

$$r = \frac{61}{28 \times 140} = 0.974.$$

Now, let us apply the short method to the same series of paired measures:

S	R	S^2	R^2	SR
4	5	16	25	20
5	8	25	64	40
6	11	36	121	66
7	11	49	121	77
8	13	64	169	104
9	18	81	324	162
10	18	100	324	180
49	84	371	1,148	649
....	....	343	1,008	588
$a=7$	$c=12$	28	140	61

$nac = 588$.
$na^2 = 343$.
$nc^1 = 1,008$.

$$r = \frac{61}{\sqrt{28 \times 140}} = 0.974.$$

It will be observed that the two methods give identical results. By using the new method it is necessary only to use a table of squares and an adding machine, if these are available, and about one-tenth of the time ordinarily used for finding the coefficient of correlation.

If it is desired to get the standard deviations of these series, they are readily obtainable, because the quantities in the denominator are the sums of the squares of the deviations of these measures from their respective averages. We have the following additional formulae:

$$b_1 = r\frac{\sigma_S}{\sigma_R} = \left[\frac{\Sigma SR - nac}{\sqrt{(\Sigma S^2 - na^2)(\Sigma R^2 - nc^2)}}\right]\left[\frac{\sqrt{\dfrac{\Sigma S^2 - na^2}{n}}}{\sqrt{\dfrac{\Sigma R^2 - nc^2}{n}}}\right]$$

$$= \frac{\Sigma SR - nac}{\Sigma R^2 - nc^2}$$

$$b_2 = r\frac{\sigma_R}{\sigma_S} = \frac{\Sigma SR - nac}{\Sigma S^2 - na^2}$$

These values in the case of the above illustrative example are 2.18 and 0.436, respectively.

For single series of independent measures, we obviously have:

$$\text{Standard deviation} = \sqrt{\frac{\Sigma m^2 - na^2}{n}},$$

$$\text{Average deviation} = \frac{n(\Sigma m_+ - \Sigma m_-) - \Sigma m(n_+ - n_-)}{n^2},$$

$$\text{Coefficient of variation} = \sqrt{\frac{n\Sigma m^2 - (\Sigma m)^2}{\Sigma m}}.$$

Where possible the solution should be in terms of class intervals rather than in that of the unit of measure.

CLIMATE AND PHOTOGRAPHY.

By H. G. CORNTHWAITE.

[Rockville, Ind., April 16, 1922.]

SYNOPSIS.

The weather or climatic element in photography is an important one, first, because of the wide variations in the strength of daylight with the time of day, season of the year, condition of the sky, with latitude, and with altitude; and, second, because of the important effects of temperature and humidity conditions have on photographic chemical processes.

Camera operators often produce inferior work in an unfamiliar climatic environment, which suggests the desirability of becoming familiar with climatic and weather conditions and their effects upon photographic work and processes.

As with many other forms of human activity, the weather or climatic element is of first importance in photographic work.

The following notes and observations are based on the writer's experience operating a camera under varying climatic conditions.

The more important climatic influences affecting photographic work may be discussed under two heads: *Intensity of sunlight,* and *Weather conditions affecting photographic chemical processes.*

INTENSITY OF SUNLIGHT.

The intensity of sunlight is perhaps the most important climatic condition affecting outdoor speed photography, as it controls the time of exposure. It varies greatly with the season of the year, the time of day, the condition of the sky (cloudiness), with latitude, and to a less degree with altitude.

The diurnal variations in the actinic (photographic) strength of daylight is well known, the light being brightest when the sun is at or near the zenith and dimming rapidly with increasing obliquity of its rays.

The seasonal variation in the strength of daylight is due to the same cause, variations from season to season in the obliquity of the sun's rays. Amateur photographers too often overlook or underestimate the effects of this seasonal variation. A bright late autumn or winter day *looks* about as bright as a similar summer day, but the photographic strength of the light is perhaps twice as great in summer as in late autumn or winter.

Of a similar character are the variations in the photographic strength of daylight due to changes in latitude, the light being strongest in the Tropics and progressively dimming poleward in each direction. Here, too, photographers often fail to make proper allowance for the wide variations in the strength of light due to the varying degree of obliquity of the sun's rays in different latitudes.

Generally speaking, the photographic strength of light increases with altitude, as the air is less dense at higher altitudes and absorbs fewer of the sun's rays, especially the short wave-length rays of greatest photographic strength. At higher altitudes there is also a greater amount of reflected light gathered by the camera lens.

The effects of cloudiness and fog in reducing the strength of daylight are too well known to require comment.

The amount and distribution of rainfall *indirectly* affect the time required for outdoor exposures, especially landscapes, as rainfall controls in large measure the growth, distribution, and density of vegetation, and the light reflected from green vegetation is of weak photographic strength. Desert scenes require much shorter exposures than views in grassy or forest areas.

Heavy rainfall has a surprising effect on photographic exposures. During a heavy tropical downpour an exposure of one-twenty-fifth second with open lens (speed F. 8) was found to be correct, *the light being actually stronger photographically during the heavy downpour than it was in densely cloudy weather without rainfall,* due to the light reflected from the falling raindrops.

Tropical daylight is perhaps twice as strong photographically as summer daylight in latitude 40 and about four times as bright as winter daylight at this latitude. This relationship, of course, does not hold true when winter landscapes are covered with a dazzlingly white blanket of snow.

The light generally is brighter in the Rocky Mountain and Pacific coast sections of the United States than in the Central and Atlantic coast sections. It is much brighter also along the seacoast than inland.

WEATHER CONDITIONS AFFECTING PHOTOGRAPHIC CHEMICAL PROCESSES.

Temperature and humidity are the important weather elements affecting photographic chemical processes. Chemical activity in developing and fixing processes is